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Exact result for the Green's function of a two-dimensional interacting electron gas with spin-orbit interaction in a perpendicular magnetic field

Chao Li

SKLSM, Institute of Semiconductors, Chinese Academy of Sciences, Beijing 100083, People's Republic of China

E-mail: lichao@semi.ac.cn

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Abstract

An exact property is established for the Green's function of a uniform two-dimensional interacting electron gas in a perpendicular magnetic field with spin-orbit interaction. It is shown that the spin-diagonal Green's function is exactly diagonal in the Landau level index even in the presence of electron-electron interactions. For the Green's function with different spin indexes, only that with adjacent Landau level indexes is non-zero. This exact result should be helpful in calculating the Green's function approximately.

1. Introduction

The Green's function approach has been widely and successfully used in solid state physics dealing with many-body systems [1–4]. In this systematic approach, the one-particle Green's function plays a central role, because one can obtain from the Green's function many system properties such as charge and spin densities, the total energy, the dispersion of elemental excitations and the linear response of the system to an externally applied perturbation. In some cases, typically in a single-particle system, one may calculate the Green's function exactly, even in closed form [5, 6]. However, in a many-body system, the particle-particle interaction cannot be handled fully and exactly in general, and one has to make approximations in the calculation of the Green's function. Because of this, exact properties of the Green's function are desired, as they can be useful in examining the validity of approximations [7, 8].

Recently, spin-orbit coupling has attracted considerable experimental and theoretical interest [9–15]. The Green's function method has been used in examining the influence of spin-orbit coupling. In a two-dimensional electron system such as a semiconductor quantum well and heterostructure, there are usually two kinds of spin-orbit coupling: the Rashba type coupling, which comes from the inversion asymmetry of

the confining potential, and the Dresselhaus type coupling, which is due to bulk inversion asymmetry [16, 17]. The Rashba type coupling is more interesting, as the strength of the Rashba spin-orbit coupling can be tuned externally by a gate voltage [18–20]. For this reason, in this paper we will limit ourselves to the Rashba spin-orbit coupling case.

The aim of the present paper is to derive an exact property for the one-particle Green's function of a two-dimensional electron gas in a perpendicular magnetic field including the influence of the Rashba spin-orbit coupling. The derived result should be helpful in the approximate calculation of the Green's function. This paper is organized as follows: in section 2, the derivation of the exact property will be presented. Then a summary is given in the last section.

2. Derivation of the exact property

Let us start by considering the following Hamiltonian:

$$H = \sum_{\sigma} \frac{1}{2m} \int d\mathbf{r} \psi_{\sigma}^{\dagger}(\mathbf{r}) \left[\mathbf{p} + \frac{e\mathbf{A}(\mathbf{r})}{c} \right]^2 \psi_{\sigma}(\mathbf{r}) + \sum_{\sigma, \sigma'} \int d\mathbf{r} \psi_{\sigma}^{\dagger}(\mathbf{r}) \left[g^* \mu_B B \frac{\sigma_z}{2} \right]_{\sigma, \sigma'} \psi_{\sigma'}(\mathbf{r})$$

$$\begin{aligned}
& + \frac{1}{2} \sum_{\sigma, \sigma'} \int d\mathbf{r} \int d\mathbf{r}' \psi_{\sigma}^{\dagger}(\mathbf{r}) \psi_{\sigma'}^{\dagger}(\mathbf{r}') V(\mathbf{r} - \mathbf{r}') \psi_{\sigma'}(\mathbf{r}') \psi_{\sigma}(\mathbf{r}) \\
& + \sum_{\sigma, \sigma'} \int d\mathbf{r} \psi_{\sigma}^{\dagger}(\mathbf{r}) \left[\alpha_R \sigma_x \left(p_y + \frac{eA_y(\mathbf{r})}{c} \right) \right. \\
& \left. - \alpha_R \sigma_y \left(p_x + \frac{eA_x(\mathbf{r})}{c} \right) \right]_{\sigma, \sigma'} \psi_{\sigma'}(\mathbf{r}), \quad (1)
\end{aligned}$$

where $V(\mathbf{r})$ is the electron–electron interaction potential and ψ is the field operator. α_R gives the strength of the Rashba interaction. g^* gives the effective Zeeman spin splitting. The Pauli matrices σ_x , σ_y and σ_z are given by [21]

$$\begin{aligned}
\sigma_x &= \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, & \sigma_y &= \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}, \\
\sigma_z &= \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}. \quad (2)
\end{aligned}$$

The coordinate of the system is chosen in such a way that $\mathbf{r} = (x, y)$ is in the xy plane. The vector potential \mathbf{A} produces a magnetic field in the z direction. In the Landau gauge, $\mathbf{A} = (0, Bx, 0)$, the single-particle states (wavefunctions) are given by

$$\begin{aligned}
\varphi_{nk}(\mathbf{r}) &= e^{iky} \alpha^{1/2} \varphi_n(\alpha x + k/\alpha), \\
\varphi_n(x) &= (\sqrt{\pi} 2^n n!)^{-1/2} e^{-x^2/2} H_n(x), \quad (3)
\end{aligned}$$

with $\alpha = (m\omega_c/\hbar)^{1/2}$, $H_n(x)$ the Hermite polynomial, $\omega_c = eB/mc$ and n the Landau level index.

Because H is time-independent, the Green's function $G_{\sigma, \sigma'}(\mathbf{r}, \mathbf{r}', t)$ can be defined as (without the loss of generality, only $t \geq 0$ needs to be considered) [1]

$$G_{\sigma, \sigma'}(\mathbf{r}, \mathbf{r}', t) = -i \text{Tr}[\rho \psi_{\sigma}(\mathbf{r}, t) \psi_{\sigma'}^{\dagger}(\mathbf{r}', 0)], \quad (4)$$

with $\rho = e^{\beta(\Omega - K)}$ the density matrix, $\psi_{\sigma}(\mathbf{r}, t) = e^{iKt/\hbar} \psi_{\sigma}(\mathbf{r}) e^{-iKt/\hbar}$, $K = H - \mu N$, $N = \sum_{\sigma} \int d\mathbf{r} \psi_{\sigma}^{\dagger}(\mathbf{r}) \psi_{\sigma}(\mathbf{r})$, the total number of particles and μ the chemical potential.

First, let us consider an unitary transformation U_1 , given by

$$U_1 = \exp \left[-\frac{i}{\hbar} \mathbf{s} \cdot \sum_{\sigma} \int d\mathbf{r} \psi_{\sigma}^{\dagger}(\mathbf{r}) \mathbf{p} \psi_{\sigma}(\mathbf{r}) \right]. \quad (5)$$

It is easy to verify that $U_1^{\dagger} = U_1^{-1}$, and that under this transformation, $U_1 \psi_{\sigma}(\mathbf{r}) U_1^{-1} = \psi_{\sigma}(\mathbf{r} + \mathbf{s})$. In deriving this result, one uses $e^A B e^{-A} = B + [A, B] + [A, [A, B]]/2! + [A, [A, [A, B]]]/3! + \dots$. Applying this transformation to H , then $H_1 = U_1 H U_1^{-1}$ can be written as

$$\begin{aligned}
H_1 &= \sum_{\sigma} \frac{1}{2m} \int d\mathbf{r} \psi_{\sigma}^{\dagger}(\mathbf{r}) \left[\mathbf{p} + \frac{e\mathbf{A}(\mathbf{r} - \mathbf{s})}{c} \right]^2 \psi_{\sigma}(\mathbf{r}) \\
& + \sum_{\sigma, \sigma'} \int d\mathbf{r} \psi_{\sigma}^{\dagger}(\mathbf{r}) \left[g^* \mu_B B \frac{\sigma_z}{2} \right]_{\sigma, \sigma'} \psi_{\sigma'}(\mathbf{r}) \\
& + \frac{1}{2} \sum_{\sigma, \sigma'} \int d\mathbf{r} \int d\mathbf{r}' \psi_{\sigma}^{\dagger}(\mathbf{r}) \psi_{\sigma'}^{\dagger}(\mathbf{r}') V(\mathbf{r} - \mathbf{r}') \psi_{\sigma'}(\mathbf{r}') \psi_{\sigma}(\mathbf{r}) \\
& + \sum_{\sigma, \sigma'} \int d\mathbf{r} \psi_{\sigma}^{\dagger}(\mathbf{r}) \left[\alpha_R \sigma_x \left(p_y + \frac{eA_y(\mathbf{r} - \mathbf{s})}{c} \right) \right. \\
& \left. - \alpha_R \sigma_y \left(p_x + \frac{eA_x(\mathbf{r} - \mathbf{s})}{c} \right) \right]_{\sigma, \sigma'} \psi_{\sigma'}(\mathbf{r}). \quad (6)
\end{aligned}$$

It is evident that H_1 differs from H only in the vector potential, i.e. $\mathbf{A} = (0, B(x - s_x), 0)$.

Next, let us examine unitary transformation U_2 , defined as

$$U_2 = \exp \left[-\sum_{\sigma} \int d\mathbf{r} \psi_{\sigma}^{\dagger}(\mathbf{r}) [i\alpha^2 s_x y] \psi_{\sigma}(\mathbf{r}) \right], \quad (7)$$

with s_x the x component of \mathbf{s} . It is easy to verify that $U_2^{\dagger} = U_2^{-1}$, and that under this transformation, $U_2 \psi_{\sigma}(\mathbf{r}) U_2^{-1} = e^{i\alpha^2 s_x y} \psi_{\sigma}(\mathbf{r})$. Applying this transformation to H_1 , one finds that $U_2 H_1 U_2^{-1} = H$ exactly. Note that $U_1 N U_1^{-1} = U_2 N U_2^{-1} = N$.

Inserting $1 = U^{-1} U$ (U denotes U_1 and U_2) into the trace calculation defining the Green's function, since the value of the trace does not change, one obtains

$$\begin{aligned}
G_{\sigma, \sigma'}(\mathbf{r}, \mathbf{r}', t) &= -i e^{i\alpha^2 s_x (y - y')} \\
&\times \text{Tr} [e^{\beta(\Omega - K)} e^{iKt/\hbar} \psi_{\sigma}(\mathbf{r} + \mathbf{s}) e^{-iKt/\hbar} \psi_{\sigma'}^{\dagger}(\mathbf{r}' + \mathbf{s})]. \quad (8)
\end{aligned}$$

Choosing $\mathbf{s} = -(\mathbf{r} + \mathbf{r}')/2$, and denoting $\mathbf{z} = \mathbf{r} - \mathbf{r}'$, one obtains

$$G_{\sigma, \sigma'}(\mathbf{r}, \mathbf{r}', t) = G_{\sigma, \sigma'}(\mathbf{z}/2, -\mathbf{z}/2, t) e^{-i\alpha^2 (x+x')(y-y')/2}. \quad (9)$$

Let us consider unitary transformation U_3 , given by

$$U_3 = \exp \left[-\sum_{\sigma} \int d\mathbf{r} \psi_{\sigma}^{\dagger}(\mathbf{r}) [-i\alpha^2 x y/2] \psi_{\sigma}(\mathbf{r}) \right]. \quad (10)$$

It is easy to verify that $U_3^{\dagger} = U_3^{-1}$, and that under this transformation, $U_3 \psi_{\sigma}(\mathbf{r}) U_3^{-1} = e^{-i\alpha^2 x y/2} \psi_{\sigma}(\mathbf{r})$. Note that $U_3 N U_3^{-1} = N$. Applying this transformation to H , one finds that $H_S = U_3 H U_3^{-1}$ differs from H only in the vector potential. In H_S , the vector potential is $\mathbf{A} = (B/2)(-y, x, 0)$, the symmetrical gauge.

Inserting $1 = U_3^{-1} U_3$ into the trace calculation defining $G_{\sigma, \sigma'}(\mathbf{r}, \mathbf{r}', t)$ above, one obtains

$$G_{\sigma, \sigma'}(\mathbf{r}, \mathbf{r}', t) = G_{\sigma, \sigma'}^{(S)}(\mathbf{z}/2, -\mathbf{z}/2, t) e^{-i\alpha^2 (x+x')(y-y')/2}, \quad (11)$$

where the superscript (S) emphasizes that $G_{\sigma, \sigma'}^{(S)}$ is calculated with Hamiltonian H_S in the symmetrical gauge.

Note that with the symmetrical gauge, the Hamiltonian H_S is invariant under the transformation of rotation about the z axis. This is because the electron–electron interaction and spin–orbit interaction are invariant under the rotation, and the kinetic energy is also invariant in the symmetrical gauge. Let us examine this in more detail. The rotation transformation about the z axis is defined as

$$U_4 = \exp \left[-\frac{i}{\hbar} \phi_0 \sum_{\sigma, \sigma'} \int d\mathbf{r} \psi_{\sigma}^{\dagger}(\mathbf{r}) [L_z + \hbar \sigma_z/2]_{\sigma, \sigma'} \psi_{\sigma'}(\mathbf{r}) \right], \quad (12)$$

with ϕ_0 the amount of rotation. One has

$$\begin{aligned}
U_4 \psi_{\uparrow}(r, \theta, \phi) U_4^{-1} &= e^{i\phi_0/2} \psi_{\uparrow}(r, \theta, \phi + \phi_0), \\
U_4 \psi_{\downarrow}(r, \theta, \phi) U_4^{-1} &= e^{-i\phi_0/2} \psi_{\downarrow}(r, \theta, \phi + \phi_0), \quad (13)
\end{aligned}$$

for spherical polar coordinates. In cylindrical coordinates, one has

$$\begin{aligned} U_4 \psi_{\uparrow}(r, \phi, z) U_4^{-1} &= e^{i\phi_0/2} \psi_{\uparrow}(r, \phi + \phi_0, z), \\ U_4 \psi_{\downarrow}(r, \phi, z) U_4^{-1} &= e^{-i\phi_0/2} \psi_{\downarrow}(r, \phi + \phi_0, z). \end{aligned} \quad (14)$$

In the first quantization form, for the symmetrical gauge case, i.e. $\mathbf{A} = (B/2)(-y, x, 0)$, the kinetic energy part of the Hamiltonian is $(\mathbf{p} + e\mathbf{A}/c)^2 = \mathbf{p}^2 + e^2\mathbf{A}^2/c^2 + (eB/c)(xp_y - yp_x)$ and $xp_y - yp_x = L_z$. In cylindrical coordinates, $L_z = (\hbar/i)\partial/\partial\phi$. Thus, the kinetic energy operator does not contain ϕ explicitly, only $\partial/\partial\phi$ is involved. It is clear that the Zeeman splitting term is invariant under the rotation transformation about the z axis, as it only involves σ_z .

Let us examine the Rashba type spin-orbit interaction next. This interaction is given by $H' = \sigma_x(p_y + eA_y/c) - \sigma_y(p_x + eA_x/c)$. In cylindrical coordinates, one has

$$\begin{aligned} H' &= \sigma_x[\sin\phi p_r + \cos\phi p_\phi + (eB/2c)r \cos\phi] \\ &\quad - \sigma_y[\cos\phi p_r - \sin\phi p_\phi - (eB/2c)r \sin\phi] \\ &= (\sin\phi\sigma_x - \cos\phi\sigma_y)p_r + (\cos\phi\sigma_x + \sin\phi\sigma_y)p_\phi \\ &\quad + (eB/2c)(\cos\phi\sigma_x + \sin\phi\sigma_y)r. \end{aligned} \quad (15)$$

Here $p_r = (\hbar/i)\partial/\partial r$ and $p_\phi = (\hbar/ir)\partial/\partial\phi$. In matrix format, one has

$$\begin{aligned} \sin\phi\sigma_x - \cos\phi\sigma_y &= \begin{pmatrix} 0 & ie^{-i\phi} \\ -ie^{i\phi} & 0 \end{pmatrix}, \\ \cos\phi\sigma_x + \sin\phi\sigma_y &= \begin{pmatrix} 0 & e^{-i\phi} \\ e^{i\phi} & 0 \end{pmatrix}. \end{aligned} \quad (16)$$

One sees that

$$\begin{aligned} e^{-i\phi_0/2\sigma_z}(\sin\phi\sigma_x - \cos\phi\sigma_y)e^{i\phi_0/2\sigma_z} &= \sin(\phi + \phi_0)\sigma_x - \cos(\phi + \phi_0)\sigma_y, \\ e^{-i\phi_0/2\sigma_z}(\cos\phi\sigma_x + \sin\phi\sigma_y)e^{i\phi_0/2\sigma_z} &= \cos(\phi + \phi_0)\sigma_x + \sin(\phi + \phi_0)\sigma_y. \end{aligned} \quad (17)$$

Note that the spin-orbit interaction term in the Hamiltonian can be written as

$$\begin{aligned} H_{so} &= \int d\mathbf{r} \begin{pmatrix} \psi_{\uparrow}(\mathbf{r}) \\ \psi_{\downarrow}(\mathbf{r}) \end{pmatrix}^\dagger \left[\begin{pmatrix} 0 & ie^{-i\phi} \\ -ie^{i\phi} & 0 \end{pmatrix} p_r \right. \\ &\quad \left. + \begin{pmatrix} 0 & e^{-i\phi} \\ e^{i\phi} & 0 \end{pmatrix} \left(p_\phi + \frac{eBr}{2c} \right) \right] \begin{pmatrix} \psi_{\uparrow}(\mathbf{r}) \\ \psi_{\downarrow}(\mathbf{r}) \end{pmatrix}. \end{aligned} \quad (18)$$

This shows that the Rashba spin-orbit interaction is indeed invariant under rotation transformation about the z axis, given by U_4 above.

Consequently, $G_{\sigma,\sigma}^{(S)}(\mathbf{r}, -\mathbf{r}, t)$, for the same spin index, is a function of $|\mathbf{r}|$ only, independent of the direction of \mathbf{r} . For different spin indexes, by inserting $U_4^\dagger U_4 = 1$ into the definition of $G_{\sigma,\sigma'}^{(S)}$, one has

$$\begin{aligned} G_{\uparrow,\downarrow}^{(S)}(r, \phi; r', \phi'; t) &= e^{i\phi_0} G_{\uparrow,\downarrow}^{(S)}(r, \phi + \phi_0; r', \phi' + \phi_0; t), \\ G_{\downarrow,\uparrow}^{(S)}(r, \phi; r', \phi'; t) &= e^{-i\phi_0} G_{\downarrow,\uparrow}^{(S)}(r, \phi + \phi_0; r', \phi' + \phi_0; t), \end{aligned} \quad (19)$$

for any ϕ_0 . This shows that

$$\begin{aligned} G_{\uparrow,\downarrow}^{(S)}(\mathbf{r}, -\mathbf{r}, t) &= e^{-i\phi} G_{\uparrow,\downarrow}^{(S)}(r, 0; r, \pi; t), \\ G_{\downarrow,\uparrow}^{(S)}(\mathbf{r}, -\mathbf{r}, t) &= e^{i\phi} G_{\downarrow,\uparrow}^{(S)}(r, 0; r, \pi; t). \end{aligned} \quad (20)$$

Now, it is ready to calculate

$$\begin{aligned} G_{\sigma,\sigma'}(nk, n'k', t) &= \int d\mathbf{r} \int d\mathbf{r}' \varphi_{nk}^*(\mathbf{r}) G_{\sigma,\sigma'}(\mathbf{r}, \mathbf{r}', t) \varphi_{n'k'}(\mathbf{r}') \\ &= \int d\mathbf{z} G_{\sigma,\sigma'}^{(S)}(\mathbf{z}/2, -\mathbf{z}/2, t) \\ &\quad \times \int d\mathbf{R} \varphi_{nk}^*(\mathbf{R} + \mathbf{z}/2) \varphi_{n'k'}(\mathbf{R} - \mathbf{z}/2) e^{-i\alpha^2 Xv}, \end{aligned} \quad (21)$$

where a change of integration variables, $\mathbf{z} = \mathbf{r} - \mathbf{r}' = (u, v)$ and $\mathbf{R} = (\mathbf{r} + \mathbf{r}')/2 = (X, Y)$, is made. Note that the domains of the \mathbf{r} and \mathbf{r}' integrals are the whole xy plane. Thus, the domains of the \mathbf{z} and \mathbf{R} integrals are also the whole xy plane.

Let us examine the \mathbf{R} integral [22, 23]:

$$\begin{aligned} I &= \int d\mathbf{R} \varphi_{nk}^*(\mathbf{R} + \mathbf{z}/2) \varphi_{n'k'}(\mathbf{R} - \mathbf{z}/2) e^{-i\alpha^2 Xv} \\ &= \alpha \int_{-\infty}^{+\infty} dX \varphi_n(\alpha(X + u/2) + k/\alpha) \\ &\quad \times \varphi_{n'}(\alpha(X - u/2) + k'/\alpha) e^{-i\alpha^2 Xv} \\ &\quad \times \int_{-\infty}^{+\infty} dY e^{-ik(Y+v/2)+ik'(Y-v/2)} \\ &= \alpha \delta_{k,k'} \int_{-\infty}^{+\infty} dX \varphi_n(\alpha(X + u/2) + k/\alpha) \\ &\quad \times \varphi_{n'}(\alpha(X - u/2) + k'/\alpha) e^{-i\alpha(\alpha X + k/\alpha)v} \\ &= \delta_{k,k'} (\pi 2^{n+n'} n!n'!)^{-1/2} \int_{-\infty}^{+\infty} dx e^{-\alpha^2 u^2/4 - x^2 - ixv} \\ &\quad \times H_n(x + \alpha u/2) H_{n'}(x - \alpha u/2) \\ &= \delta_{k,k'} (-1)^{(n'-n+|n'-n|)/2} H_{n,n'}(\alpha^2 z^2/2) e^{i(n'-n)\phi}, \end{aligned} \quad (22)$$

where ϕ is introduced via $u = z \cos(\phi)$, $v = z \sin(\phi)$. $H_{n,n'}(x) = (n!/n'!)^{1/2} e^{-x/2} x^{|n-n'|/2} L_n^{|n-n'|}(x)$, for $n \leq n'$. For $n \geq n'$, $H_{n,n'}(x)$ results from an interchange of n and n' in the above expression.

Finally, let us examine $G_{\sigma,\sigma'}(nk, n'k', t)$, which can be written as

$$\begin{aligned} G_{\sigma,\sigma'}(nk, n'k', t) &= \int d\mathbf{z} G_{\sigma,\sigma'}^{(S)}(\mathbf{z}/2, -\mathbf{z}/2, t) \\ &\quad \times \delta_{k,k'} (-1)^{(n'-n+|n'-n|)/2} H_{n,n'}(\alpha^2 z^2/2) e^{i(n'-n)\phi}. \end{aligned} \quad (23)$$

Since $G_{\sigma,\sigma'}^{(S)}(\mathbf{z}/2, -\mathbf{z}/2, t)$ is independent of the direction of \mathbf{z} , the ϕ integral immediately leads to the factor $\delta_{n,n'}$. Therefore

$$G_{\sigma,\sigma}(nk, n'k', t) = G_{\sigma,\sigma}(nk, nk, t) \delta_{k,k'} \delta_{n,n'}, \quad (24)$$

i.e. the spin-diagonal Green's function is exactly diagonal in the Landau level index, even in the presence of electron-electron interactions. For different spin indexes, $G_{\uparrow,\downarrow}$ will have a factor $\delta_{n',n+1}$ and $G_{\downarrow,\uparrow}$ will have a factor $\delta_{n,n'+1}$. That is

$$\begin{aligned} G_{\uparrow,\downarrow}(nk, n'k', t) &= G_{\uparrow,\downarrow}(nk, n'k, t) \delta_{k,k'} \delta_{n+1,n'}, \\ G_{\downarrow,\uparrow}(nk, n'k', t) &= G_{\downarrow,\uparrow}(nk, n'k, t) \delta_{k,k'} \delta_{n,n'+1}. \end{aligned} \quad (25)$$

Note that $G_{\sigma,\sigma'}(nk, n'k', t)$ is actually k independent. The above equations are the central results of the present paper.

Finally, let us briefly consider a different type of spin-orbit interaction, which is given by $H' = \sigma_x(p_x + eA_x/c) - \sigma_y(p_y + eA_y/c)$ in the first quantization form. This is the Dresselhaus spin-orbit interaction approximated for the two-dimensional case. For the case of the symmetrical gauge, this spin-orbit interaction can be written as in matrix form as

$$\begin{aligned} H' &= \sigma_x \left(\cos \phi p_r - \sin \phi p_\phi - \frac{eBr}{2c} \sin \phi \right) \\ &\quad - \sigma_y \left(\sin \phi p_r + \cos \phi p_\phi + \frac{eBr}{2c} \cos \phi \right) \\ &= (\cos \phi \sigma_x - \sin \phi \sigma_y) p_r \\ &\quad - (\sin \phi \sigma_x + \cos \phi \sigma_y) \left(p_\phi + \frac{eBr}{2c} \right) \\ &= \begin{pmatrix} 0 & e^{i\phi} \\ e^{-i\phi} & 0 \end{pmatrix} p_r - \begin{pmatrix} 0 & -ie^{i\phi} \\ ie^{-i\phi} & 0 \end{pmatrix} \left(p_\phi + \frac{eBr}{2c} \right). \end{aligned} \quad (26)$$

One sees that

$$\begin{aligned} e^{i\phi_0/2\sigma_z} (\cos \phi \sigma_x - \sin \phi \sigma_y) e^{-i\phi_0/2\sigma_z} \\ &= \cos(\phi + \phi_0) \sigma_x - \sin(\phi + \phi_0) \sigma_y, \\ e^{i\phi_0/2\sigma_z} (\sin \phi \sigma_x + \cos \phi \sigma_y) e^{-i\phi_0/2\sigma_z} \\ &= \sin(\phi + \phi_0) \sigma_x + \cos(\phi + \phi_0) \sigma_y. \end{aligned} \quad (27)$$

This indicates that instead of defining U_4 previously as

$$\begin{aligned} U_4 &= \exp \left[-\frac{i}{\hbar} \phi_0 \sum_{\sigma} \int d\mathbf{r} \psi_{\sigma}^{\dagger}(\mathbf{r}) L_z \psi_{\sigma}(\mathbf{r}) \right] \\ &\quad \times \exp \left[-\frac{i}{\hbar} \phi_0 \sum_{\sigma,\sigma'} \int d\mathbf{r} \psi_{\sigma}^{\dagger}(\mathbf{r}) [\hbar\sigma_z/2]_{\sigma,\sigma'} \psi_{\sigma'}(\mathbf{r}) \right], \end{aligned} \quad (28)$$

now one should define U_4 as follows, with a different transformation for the spin component:

$$\begin{aligned} U_4 &= \exp \left[-\frac{i}{\hbar} \phi_0 \sum_{\sigma} \int d\mathbf{r} \psi_{\sigma}^{\dagger}(\mathbf{r}) L_z \psi_{\sigma}(\mathbf{r}) \right] \\ &\quad \times \exp \left[\frac{i}{\hbar} \phi_0 \sum_{\sigma,\sigma'} \int d\mathbf{r} \psi_{\sigma}^{\dagger}(\mathbf{r}) [\hbar\sigma_z/2]_{\sigma,\sigma'} \psi_{\sigma'}(\mathbf{r}) \right]. \end{aligned} \quad (29)$$

One finds that, in cylindrical coordinates,

$$\begin{aligned} U_4 \psi_{\uparrow}(r, \phi, z) U_4^{-1} &= e^{-i\phi_0/2} \psi_{\uparrow}(r, \phi + \phi_0, z), \\ U_4 \psi_{\downarrow}(r, \phi, z) U_4^{-1} &= e^{i\phi_0/2} \psi_{\downarrow}(r, \phi + \phi_0, z). \end{aligned} \quad (30)$$

With this newly introduced unitary transformation, one finds that in a system with Dresselhaus type spin-orbit coupling, the Green's function has a similar property as that for a system with the Rashba spin-orbit interaction. The Green's function will have a factor $\delta_{n,n'}$, $\delta_{n+1,n'}$ or $\delta_{n,n'+1}$, depending on the spin index involved.

3. Summary

The exact property of the Green's function of a two-dimensional interacting electron gas in a perpendicular magnetic field with spin-orbit interaction is derived. By assuming that the system is uniform and rotationally invariant, we obtain some exact and meaningful properties of the Green's function. This exact result should be helpful in evaluating the Green's function approximately.

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